Phys 272 Final Exam

Time limit: 120 minutes

Each question worths 10 points.

Constants: $e = 1.6 \times 10^{-19}C$, $\epsilon_0 = 8.85 \times 10^{-12}C^2N^{-1}m^{-2}$, $\mu_0 = 4\pi \times 10^{-7}Tm/A$, mass of proton $m_p = 1.67 \times 10^{-27}kg$, mass of electron $m_e = 9.11 \times 10^{-31}kg$.

Resistivity: Aluminum $2.65 \times 10^{-8}\Omega m$, Copper $1.68 \times 10^{-8}\Omega m$, Silver $1.59 \times 10^{-8}\Omega m$, Iron $9.71 \times 10^{-8}\Omega m$.

Density: Water $\rho_{water} = 10^3 kg/m^3$, Gold $\rho_{gold} = 19.32 \times 10^3 kg/m^3$.

1. In 1993 the radius of Hurricane Emily was about $350km$. The wind speed near the center ("eye") of the hurricane, whose radius was about $30km$, reached about $200km/h$. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. (a) Estimate the wind speed at the rim of the hurricane. (b) Estimate the pressure difference at the earth's surface between the eye and the rim. (The density of air at $1atm, 20°C$ is $1.20kg/m^3$.) (c) Is the pressure greater at the eye or at the rim? (d) If the kinetic energy of the swirling air in the eye could be converted completely to gravitational potential energy, how high would the air go?

2. The voltage output of an ac power supply is given by $V_d(t) = V_p \sin(\omega t)$. The calculations below could be done either with calculus or complex numbers, choose whatever method you prefer, but do not simply write the final answer without intermediate steps, and your final answer should be real (as opposed to complex). (a) What is the current $I(t)$ when this power supply is connected to a capacitor with capacitance C (i.e. a purely capacitive circuit)? Note that we want the current as a function of time, not just its peak value. (b) If the capacitor is now replaced by an inductor with inductance L (so it is now a purely inductive circuit), what would be the current $I(t)$? (c) Suppose the power supply is now connected in an RLC circuit (all components in series). What is the peak voltage V_{Cp} across the capacitor? (d) Give the value of V_{Cp} in the limit of: (i) $\omega \to 0$; (ii) $\omega \to \infty$.

3. Figure 1 shows two solenoids that are wound on a common cylinder. We will take the cylinder to have radius ρ and length L. Assume that the solenoid is much longer than its radius so the equation $B = \mu_0 n I$ is valid. We will consider the field that arises from solenoid 1, which has n_1 turns per unit length. The magnetic field due to solenoid 1 passes (entirely, in this case) through solenoid 2, which has n_2 turns per unit length. (a) What is the flux $\Phi_1(t)$ generated by solenoid 1's magnetic field through a single turn of solenoid 2? Express your answer in terms of $I_1(t)$ as well as the other variables introduced in the question. (b) Now find the emf $\mathcal{E}_2(t)$ induced across the entirety of solenoid 2 by the change in current in solenoid 1. Remember that both solenoids have length L. Express your answer in terms of $\frac{dI_1(t)}{dt}, n_1, n_2$, other parameters given in the introduction, and any relevant constants. (c) Use your results to find the mutual inductance of the system, in terms of the variables introduced in the question.

4. (a) Figure 2a shows a magnet near a loop of coil. What is the direction of the induced current if the magnet is moving toward the coil? [Clockwise or counterclockwise as viewed from the left.] (b) What if the magnet is moving away? (c) Figure 2c shows the direction of the induced E field. Is the magnetic field in the figure increasing or decreasing? (d) Figure 3 shows five particles entering a region of magnetic field. Which of the particles are negatively charged?

Figure 1: Question 3

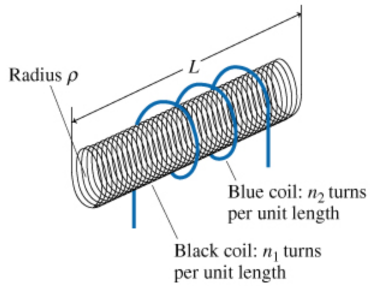


Figure 2: Question 4(a), (b), (c)

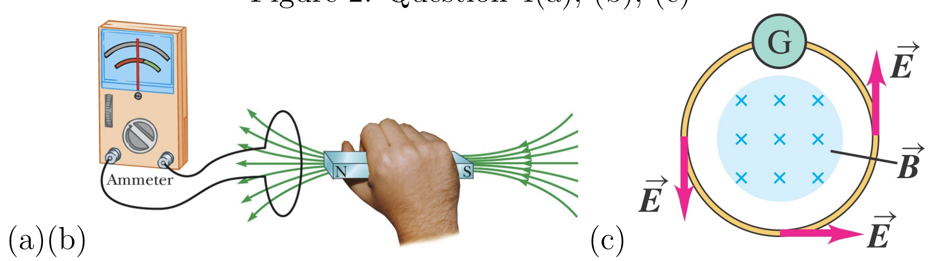
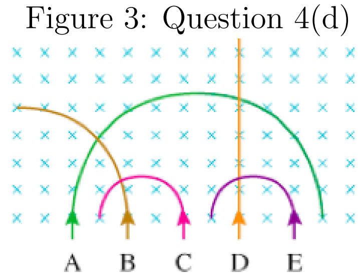
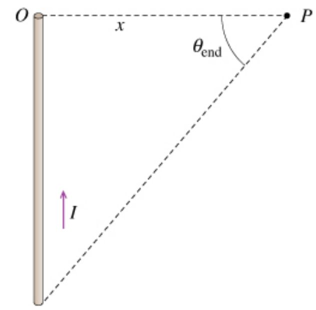


Figure 3: Question 4(d)



5. (a) State the Biot-Savart Law. (b) Figure 4 shows a finite (*not* infinite) wire carrying a current I. Using Biot-Savart Law, find the magnetic field at point P in terms of $I, x, \mu_0, \theta_{end}$. [You will get no point if you use Ampere's Law.]

Figure 4: Question 5(b)



6. (a) Figure 5 shows a battery $\mathcal{E} = 18V$ connected to the resistors $R_1 = 2\Omega$, $R_2 = 1\Omega$, $R_3 = 3\Omega$ and $R_4 = 4\Omega$. Find the current through each of the resistors (i.e. find I_1, I_2, I_3 and I_4). (b) When switch S in Figure 6 is open, the voltmeter V of the battery reads $3.07V$. When the switch is closed, the voltmeter reading drops to $3.00V$, and the ammeter A reads $1.69A$. Assume that the two meters are ideal, so they do not affect the circuit. (i) Find the emf $\mathcal{E}$ of the battery. (ii) Find the values of r and R.

Figure 5: Question 6(a)

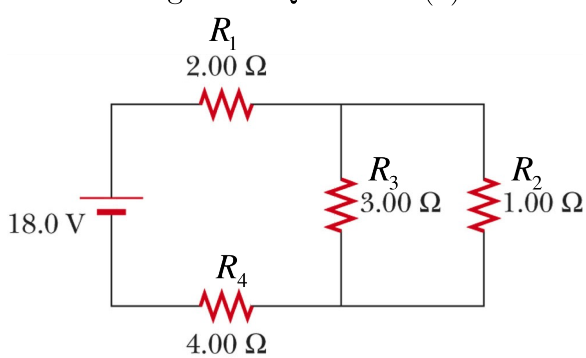
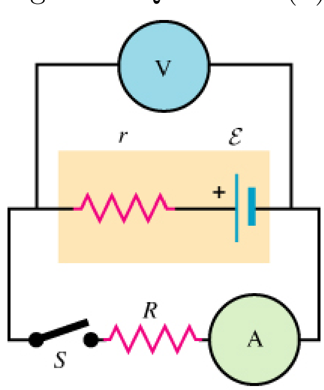


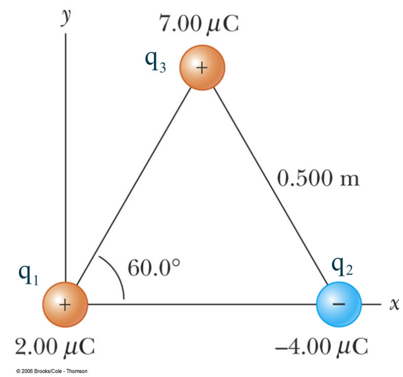
Figure 6: Question 6(b)



7. A spherical capacitor is formed from two concentric spherical conducting shells separated by vacuum. The inner sphere has radius $a = 10.0cm$, and outer sphere has radius $b = 12.0cm$. The capacitor has been charged and the magnitude of the charge on each sphere is $3.0nC$. (a) Apply Gauss' law to find the magnitude of the electric field between the spheres at radius r, where $a < r < b$. (b) Apply Gauss' law to find the magnitude of the electric field outside the outer sphere. (c) Find the capacitance of the capacitor.

8. Figure 7 shows three charges ($q_1 = 2\mu C, q_2 = -4\mu C$ and $q_3 = 7\mu C$) on an equilateral triangle (each side measures $0.5m$). (a) Find the magnitude of the electric field at the origin due to q_2 and q_3. (b) What is the total potential energy of the system? (c) How much work is done by the electric field if the charge q_1 is moved from its current location (at the origin) to infinity? Consider the sign carefully, you will receive no credit for this part for writing the wrong sign.

Figure 7: Question 8



9. A $q_1 = 2\mu C$ point charge ($1\mu = 10^{-6}$) is located at the origin, and a second point charge of $q_2 = -6\mu C$ is located on the y axis at the position $(0, 3.00)m$ as in Figure 8. (a) Find the electric field vector and also its magnitude at point P due to q_1 and q_2. (b) What is the force vector on a charge $q_3 = 3\mu C$ at point P?

Figure 8: Question 9